Topic: Period and amplitude

Question: What are the amplitude and period of the function?

$$-5\cos\frac{\theta}{7}$$

Answer choices:

A The amplitude of the function $-5\cos(\theta/7)$ is 5, and its period is 14π.

B The amplitude of the function $-5\cos(\theta/7)$ is -5, and its period is $2\pi/7$.

C The amplitude of the function $-5\cos(\theta/7)$ is 5, and its period is 2π.

D The amplitude of the function $-5\cos(\theta/7)$ is -5, and its period is 7π.

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Solution: A

The function $-5\cos(\theta/7)$ is in the form $a\cos(b\theta)$ where $a = -5$ and $b = 1/7$. Since $a = -5$, the amplitude of this function is

$$|a| = |-5| = 5$$

Since b is positive and the period of the "basic" cosine function ($\cos\theta$) is 2π, the period of the function $-5\cos(\theta/7)$ is

$$\frac{2\pi}{b} = \frac{2\pi}{\left(\frac{1}{7}\right)} = 2\pi\left(\frac{7}{1}\right) = 14\pi$$

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Topic: Period and amplitude

Question: What are the amplitude and period of the function?

$$6\csc(-2.9\theta)$$

Answer choices:

A The amplitude is -6, and its period is 2.9π.

B The amplitude is 6, and its period is 5.8π.

C The amplitude is undefined, and its period is $20\pi/29$.

D The amplitude is undefined, and its period is $\pi/2.9$.

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Solution: C

By one of the reciprocal identities,

$$\csc(-2.9\theta) = \frac{1}{\sin(-2.9\theta)}$$

And by the odd identity for the sine function,

$$\sin(-2.9\theta) = -\sin(2.9\theta)$$

Therefore,

$$\csc(-2.9\theta) = \frac{1}{-\sin(2.9\theta)} = -\frac{1}{\sin(2.9\theta)} = -\csc(2.9\theta)$$

Therefore, the function $6\csc(-2.9\theta)$ can be written as $-6\csc(2.9\theta)$, so it's in the form $a\cos(b\theta)$ with $a = -6$ and $b = 2.9$. Since we're dealing with a cosecant function, we can't consider the amplitude. Since b is positive and the period of the "basic" cosecant function ($\csc\theta$) is 2π, the period of the function $-6\csc(2.9\theta)$ is

$$\frac{2\pi}{b} = \frac{2\pi}{2.9}$$

Now note that

$$2.9 = 2 + 0.9 = 2 + \frac{9}{10} = \frac{10(2) + 9}{10} = \frac{29}{10}$$

Therefore, the period of the function $-6\csc(2.9\theta)$ is

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$$\frac{2\pi}{\left(\frac{29}{10}\right)} = 2\pi\left(\frac{10}{29}\right) = \frac{20\pi}{29}$$

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Topic: Period and amplitude

Question: Which of the following statements is false?

Answer choices:

A The period of the function $-\pi \sin(\pi^2\theta)$ is $2/\pi$.

B The period of the function $\pi^3 \tan(4\theta/3)$ is $3\pi/4$.

C The period of the function $-1.7\left[\dfrac{\sec\left(\frac{\theta}{3}\right)}{\pi}\right]$ is 6π.

D The period of the function $2\cot(4\pi\theta/3)$ is $3/2$.

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Solution: D

First, we'll show that the statement in answer choice D is false.

Note that the function

$$2\cot\left(\frac{4\pi\theta}{3}\right)$$

is in the form $a\cot(b\theta)$ with $a = 2$ and $b = 4\pi/3$. Since b is positive and the period of the "basic" cotangent function ($\cot\theta$) is π, the period of the function

$$2\cot\left(\frac{4\pi\theta}{3}\right)$$

is

$$\frac{\pi}{b} = \frac{\pi}{\left(\frac{4\pi}{3}\right)} = \pi\left(\frac{3}{4\pi}\right) = \frac{3}{4} \neq \frac{3}{2}$$

Now we'll show that the other three statements are true.

The function in answer choice A is $-\pi\sin(\pi^2\theta)$, which is in the form $a\sin(b\theta)$ with $a = -\pi$ and $b = \pi^2$. Since b is positive and the period of the "basic" sine function ($\sin\theta$) is 2π, the period of the function $-\pi\sin(\pi^2\theta)$ is

$$\frac{2\pi}{b} = \frac{2\pi}{\pi^2} = \frac{2}{\pi}$$

The function in answer choice B is

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$$\pi^3 \tan\left(\frac{4\theta}{3}\right)$$

This is in the form $a\tan(b\theta)$ with $a = \pi^3$ and $b = 4/3$. Since b is positive and the period of the "basic" tangent function ($\tan\theta$) is π, the period of the function

$$\pi^3 \tan\left(\frac{4\theta}{3}\right)$$

is

$$\frac{\pi}{b} = \frac{\pi}{\left(\frac{4}{3}\right)} = \pi\left(\frac{3}{4}\right) = \frac{3\pi}{4}$$

The function in answer choice C is

$$-1.7\left[\frac{\sec\left(\frac{\theta}{3}\right)}{\pi}\right]$$

This is in the form $a\sec(b\theta)$ with $a = -1.7/\pi$ and $b = 1/3$. The period of the "basic" secant function ($\sec\theta$) is 2π, so the period of the function

$$-1.7\left[\frac{\sec\left(\frac{\theta}{3}\right)}{\pi}\right]$$

is

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$$\frac{2\pi}{b} = \frac{2\pi}{\left(\frac{1}{3}\right)} = 2\pi\left(\frac{3}{1}\right) = 6\pi$$

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